MATHEMATICS

Paper 9709/12
Pure Mathematics

Key messages

Candidates are advised to look for all the solutions to equations, especially negative square roots and possible zeros.
The instructions for the examination clearly state that all necessary working must be shown. This applies to the solution of equations and substitution of limits as well as questions involving arithmetic processes.
Questions 7 and **8(b)** demonstrate the importance of using a higher level of accuracy in intermediate calculations compared to the accuracy required in the final answer.

General comments

Problems involving trigonometric equations require all the solutions in a given range and candidates must be careful not to include extra incorrect solutions in the given range to gain full marks.
When a question gives the answer, where candidates are commonly asked to 'show' this result it is important all stages in the working must be clearly shown.

Comments on specific questions

Question 1

The need to differentiate $f'(x)$ was well understood and many completely correct expressions for $f'(x)$ were seen. It was sufficient to state that $f'(x) < 0$ and that $f'(x)$ was a decreasing function to gain the third mark.

Question 2

In order to fully describe the two transformations, it was necessary to state the type of each of them and not their effect on the coordinates of $f(x)$. The best answers referred to the movement relative to the axes rather than vague terms such as up and down or horizontal and vertical.

Question 3

The significance of rotation about the y-axis was appreciated by many candidates who successfully found and integrated x^2 with respect to y. The substitution of the limits was usually clearly shown and few answers omitted π.

Question 4

There were many completely correct answers to this question. The use of the chain rule with the correct interpretation of the given information was widely seen. The required algebra was mostly carried out very efficiently.

Question 5

When candidates correctly multiplied by the denominator as a first stage followed by the use of $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ fewer errors were observed in the working than when other attempts were made. Factorisation of the resulting expression occasionally led to both answers. The majority of candidates found one solution correctly but did not consider the solution from $\sin\theta = 0$.

Question 6

(a) Selection of the appropriate term without completing the whole expansion was often seen. Those candidates who were careful in their use of brackets mostly went on to find the required answers. Some candidates did not identify all 'possible values' as stated in the question and did not find the negative solution.

(b) The selection of the required term proved to be more challenging in this part, however many correct answers were seen. Although only the coefficient was required, answers which included the correct power of x were able to gain the marks.

Question 7

This question was generally well answered with very few candidates confusing radians and degrees in their use of trigonometry. The formulae for the area of a triangle and a sector area were well understood and applied correctly. Many solutions featured intermediate answers to their calculations expressed to more than four decimal places and these led to the correct three significant figure accuracy in the final answer. Most answers were well set out with the required areas clearly described enabling ease in following the logic of the solutions.

Question 8

(a) This part was almost always answered correctly.

(b) Candidates who identified that the n^{th} terms of an arithmetic progression and a geometric progression were required usually quoted and used the correct formulae. The final percentage calculation was generally set up correctly. Correct rounding of the final answer was a feature of the best answers.

Question 9

(a) For most candidates, completing the square was well understood and many gained full marks on this part.

(b) The basic method to find an inverse was usually applied clearly with few algebraic errors. Those candidates who realised the range of their inverse would have to be the given domain, $x \leqslant -4$, were able to deduce that only the negative square root was valid in their expression for the inverse. The correctly expressed domain was seen in some of the best answers.

(c) The formation of the composite function was more straightforward for those who used their completed square form rather than the original function in **part (a)**. The correct resulting quadratic equation was often seen. Only candidates who considered the original domain were able to select the correct root of the equation.

(d) The use of $2k - 3 = -4$ at the maximum value of k was occasionally seen.

Question 10

(a) Many candidates realised that the given expression for the gradient should be set to zero and that the positive solution to the quadratic equation formed from this gave the required value of a. To gain full credit it was essential that all steps in the working out were shown and the required value was stated as a and not x.

(b) Almost all attempts involved finding the second derivative and its sign at the turning point. It was noted that the differentiation of $2(x+3)^{\frac{1}{2}}$ was correct more often than the differentiation of x. A few candidates produced a clearly justified conclusion based on the use of the change of sign of the first derivative around the stationary point.

(c) There were many completely correct solutions to this part. The requirements to integrate the gradient equation and use the point found in **part (a)** were well understood. The integration of $2(x+3)^{\frac{1}{2}}$ and the integration of x led to similar numbers of errors. To gain full marks it was necessary to fully state the final answer.

Question 11

(a) The majority of candidates realised the equation could be solved as a quadratic in tan x and those who showed the appropriate working often went on to gain full marks. Some answers did not include the solution from the negative value of tan x.

(b) The use of the discriminant set as less than zero was seen in the majority of responses and was often followed by correct consideration of the negative terms to reach the required inequality.

(c) Those candidates with a good understanding of the tangent graph realised that an extra solution in the range could only come from the solution of tan $x = 0$. Most then selected the appropriate value of k and went on to complete the solution.

Question 12

(a) The most successful answers to this part used the mid-point of the diameter and half the diameter length in a general equation of the circle. Those who confused the diameter with the radius were able to gain some credit when this error was repeated in **part (b)**.

(b) Finding the centre of the translated circle and use of the radius from **part (a)** in a general equation provided a quick route to the solution for many candidates.

(c) Two methods were used successfully in this part. Equating the answers from **parts (a)** and **(b)** proved the most popular. Finding the gradient and mid-point of the line of centres of the two circles then going on to find the equation of the perpendicular bisector was also regularly seen.

(d) Substitution of the given result from **part (c)** into either of the answers to **parts (a)** or **(b)** led quickly to the given result. Those who used this method were usually able to deal with the required algebra. Those who chose to eliminate y from the two circle equations needed to demonstrate very good algebraic skills to reach the given result.

MATHEMATICS

Paper 9709/22
Pure Mathematics

Key messages

It is essential that candidates read each question carefully and ensure that they have fully met the demands of that question. It is also essential that final answers are given to the correct level of accuracy, which is 3 significant figures as stated in the rubric, unless stated otherwise in the question itself.

General comments

In general candidates produced clear and well set out responses, most showing a good understanding of the syllabus objectives. Techniques used were usually appropriate and applied correctly. There appeared to be no issues with time and no issues arising from lack of space in which the responses were to be written.

Comments on specific questions

Question 1

Most candidates used a correct expression for $\sin(\theta + 30^\circ)$ and then went on to simplify the given equation to obtain an expression for $\tan\theta$. Most candidates offered fully correct solutions to this question.

Question 2

(a) It was essential that candidates show the algebraic long division essential to this part of the question as clearly as possible. A penultimate line of $-3x^2 - 15x - 18$ followed by a final line of 18, was required to gain full marks. Most were able to gain these marks.

(b) Candidates should always take note of the word 'Hence'. This implies that the work from the previous part must be used. It was essential that candidates made the link between the equation to solve and the work they had just completed in **part (a)**. This could be implied by, $(x^2 + 5x + 6)(4x - 3) + 18 - 18 = 0$, or equivalent. Marks were available if an error had been made in the quotient from **part (a)**. An answer giving just the solutions with no supporting work obtained no marks. An answer left in terms of factors gained only one possible mark as the question specified the solution of the given equation.

Question 3

Most candidates realised that the integral was in the form of a logarithm, the hint being that the integrand was equated to $\ln\frac{7}{2}$. Completely correct solutions were common, with candidates showing sufficient working and well set out solutions.

Question 4

It was essential that candidates realise that implicit differentiation was involved before any marks could be awarded. Correct implicit differentiation of both y^2 and $4\ln(2y+3)$ with respect to *x* was common. There were the occasional slips in the subsequent evaluation of $\frac{dy}{dx}$ at the given point, but most candidates were able to score reasonably well in this question.

Question 5

(a) A correctly shaped pair of modulus graphs were obtained by the majority of candidates. This is an example of where candidates did not check the demands of the question. They were asked to give, in terms of *k*, the coordinates of the points where each graph meets the axes. Some candidates did not do this. Either four sets of coordinates or the four intercepts marked on the axes were acceptable.

(b) Most candidates realised that they had to solve $|x+2k| = |2x-3k|$. Most chose to square each side of the equation and solve the resulting quadratic equation. Solutions involving the formation of two linear equations were less common. Some candidates did not obtain full marks as they omitted to find the corresponding *y* values associated with the *x* values they had just found. Another example of not checking that the demands of the question are met fully.

(c) A more challenging part of the question, candidates were expected to equate their larger value of *x* from **part (b)** to 2^t and then rearrange to obtain *t* in terms of *k*. Few completely correct solutions were seen.

Question 6

(a) Differentiation using the product rule was attempted by most candidates. Most then equated this to 15 and attempted, with varying levels of success, to rearrange the resulting equation to the given form. It is essential in questions of this type, that full working is shown to justify the given result.

(b) Most candidates realised that they had to consider, for example, $f(x) = x - \sqrt{\frac{75e^{-0.2x}}{15+x}}$ or equivalent, for the values $x = 1.7$ and $x = 1.8$. Most candidates did this and evaluated these expressions correctly, went on to make a correct conclusion.

(c) It was essential that candidates use the correct level of accuracy as required in this part of the question. Most candidates were able to start the iteration process using an appropriate starting value. Most gave their iterations to the required level of 6 significant figures. However, many candidates did not write down their final iteration which would then lead to justify their answer. Many candidates also did not give their final answer to 4 significant figures as required.

Question 7

(a) Having realised that they needed to solve the equation $4\sin^2 x + 8\sin x + 3 = 0$, usually by factorisation, some candidates, having obtained the correct result $\sin x = -\frac{1}{2}$, were unable to obtain the correct value for *x*.

(b) Most made a correct attempt to differentiate the given equation. Use of either the chain rule or an attempt at use of the double angle formula followed by differentiation was acceptable. Most candidates recognised the need to give their answer in an exact form. Few decimal answers were seen.

(c) Most candidates gave their final answer in an exact form. To gain any marks in this part of the question, it was essential that an attempt at the use of the appropriate double angle formula be made. Provided this was done, most candidates made a reasonable attempt at the integration of each term and attempted to apply the limits of 0 and their value from **part (a)**. In some cases errors in substitution, evaluation or an incorrect upper limit, meant that few correct final answers were seen.

MATHEMATICS

Paper 9709/32
Pure Mathematics

Key messages

- Candidates should ensure that they know what is expected when asked to sketch of a graph, which was needed for **Question 1(a)** and **Question 3(a)**.
- Candidates are also expected to understand what is meant by tangent parallel to an axis, which was seen in **Question 7(b)**.
- It is important for candidates to use correct mathematical notation for the vector equation of a line, as required in **Question 8(b)**.
- It is essential for candidates to present all detailed working, especially when the answer can also be obtained using a calculator. This was particularly notable for **Question 10(a)**.
- Candidates should also be reminded that that scales on an Argand diagram must always be the same, essential for **Question 10(b)(i)**.

General comments

Many candidates produced excellent solutions to **Question 9** on partial fractions and **Question 7(a)** on implicit differentiation. However, few good attempts at finding the perpendicular vector in **Question 8(c)** and the complex solutions of the equations in **Question 10(a)** were seen.

Questions that candidates answered most successfully were **Question 1(b), 2, 3(b)**, **3(c)**, the initial part of **Question 4** involving integration, **Question 7(a), 8(a)** and **10(b)(i)**. The questions that candidates generally found more challenging were **Question 1**, **3(a)**, **5**, **6(a)**, **8(b)**, **8(c)**, **10(a)**, **10(b)(ii)**.

Candidates should be reminded to ensure that their work is clearly presented and, in particular, should avoid changing signs by writing over the top of their original working. Where corrections or replacements have been made candidates must ensure that what is their final solution is clearly identifiable and legible. Greater care is also required when removing brackets when there is both a negative sign inside and outside the bracket. Candidates should also be reminded to work systematically down the page to demonstrate a clear order to their method.

Comments on specific questions

Question 1

(a) There were some excellent solutions to this question, however, there were also many incorrect solutions, for example a single straight line was very common. Some candidates did not show the crucial point $x = 2$ on the x-axis. Simply plotting points and joining them all up is not the best approach to adopt, as often, unless done extremely carefully it leads to something that is not an exact straight line. Many graphs stopped at the y-axis, or likewise at $x = 2$. Others had scales on both axes, but with their graph clearly cutting the y-axis at a value other than $y = 2$. A correct graph should have been a symmetrical V shape with its base at (2, 0), the coordinates of the intercepts on each axis stated, together with a domain of at least $-1 \leqslant x \leqslant 5$.

(b) Many candidates correctly established the critical points, however few continued to show $x > \frac{3}{2}$ was their final answer. There were several incorrect quadratic equations amongst those candidates that adopted this approach, some due to not squaring the linear terms correctly. Many others who took this approach made the error of only squaring the modulus side of the equation.

Question 2

A common error seen in this question was for candidates not to reject the negative root. The other common error amongst candidates that established the correct quadratic equation was that they did not give their answer in exact form, as stated in the question. Many candidates did not deal with the power ln law correctly, with the addition of ln terms resulting in $3 + (2x + 5)$ instead of $3(2x + 5)$.

Question 3

(a) In many cases, candidates found the sketching of $y = \sec x$ challenging. Few candidates were able to produce a sketch representing the function, and amongst candidates who sketched a nearly correct graph most either stopped at $\left(\frac{\pi}{3}, 2\right)$ or continued beyond $x = \frac{\pi}{2}$. The most straightforward way to produce a good sketch is to sketch the well-known graph of $y = \cos x$ and then reciprocate this graph. The final graph should show (0, 1), together with the asymptote at $x = \frac{\pi}{2}$. The straight-line graph is more straightforward to sketch; however, this was often seen to cross the x-axis in the region of $x = \frac{\pi}{2}$. Although as only the region $0 \leqslant x \leqslant \frac{\pi}{2}$ was requested, if the candidate opts to show a greater region than this then the line must not be seen to cross the x-axis at an incorrect value in order to gain the accuracy mark.

(b) Most candidates answered this question part well.

(c) Most candidates found this part very straight forward, although some candidates incorrectly worked with their calculator in degrees mode.

Question 4

Several different errors were seen in candidate responses to this question. Some candidates were able to correctly establish just the first integration by parts before making subsequent errors. A common error for these candidates was to include $\ln(\sin x)$ instead of $\ln(\cos x)$. Other candidates who managed both integrations by parts often made errors with their limits, including working in decimals. Some other candidates were able to complete the first parts successfully including substituting their limits, however then made errors in combining terms. Of the candidates who made these errors, the most common was the incorrect removal of brackets which often made it difficult to establish whether the required ln law had been applied correctly. It is important for candidates to keep their values exact, as stated in the question, and show clear application of their limits when completing the integration. For candidates who established the correct decimal answer of 0.962 a special case B1 mark was available.

Question 5

(a) The candidates who commenced with a single fraction usually gained all the marks efficiently. Those who expanded $\cos 3x$ and $\sin 3x$ found that more algebra was needed, especially if they continued their working for a few steps before opting for a single fraction. Typically, candidates who did more working prior to finding a common denominator found it challenging to be able to establish the stated result.

(b) Most candidates identified the need to use the result from **(a)**, and successfully established at least one of the answers. Candidates who opted to use the double angle and then solve the resulting quadratic equation were usually more successful than those who remained in $\tan 2x$. There were two main reasons for this; candidates had no need to divide by 2 at the appropriate place and further to this they did not have the problem of needing to understand the difference of inverse trigonometrical functions and the reciprocal of trigonometrical functions. Another error made was to use degrees instead of radians and another being to not give answers to 3 significant figures.

Question 6

(a) Whilst most candidates could separate variables correctly, few were able to make much progress other than possibly the correct integration of e^{-x}. Candidates who did manage to establish $a\tan^{-1}(by)$ rarely obtained $b = 2$ and almost never $a = \frac{1}{2}$.

(b) This solution was a follow through from **(a)**, however there was the necessity that their expression from **(a)** contained an e^{-x} term and tended to a finite limit. Most candidates found this challenging as a result of errors in **(a)**, for the few candidates who had made a reasonable attempt at **(a)** this mark was often successfully gained.

Question 7

(a) Most candidates performed their implicit differentiation correctly and it was usually only a sign error in the manipulation to the answer given. Candidates who made errors often did not show their factorising of the $\frac{dy}{dx}$ terms or sometimes lacked detail of the derivative on the right side of the equation within an equation structure.

(b) Many candidates incorrectly interpreted that the 'tangent parallel to the y-axis' meant $\frac{dy}{dx}$ was either zero or one. Candidates that did realise this required the use of $\frac{dx}{dy}$ to be zero then often omitted to consider the solution arising from the equation $y = 0$.

Question 8

(a) Most candidates identified what was required to establish the vectors $\overrightarrow{OM}$ and $\overrightarrow{MN}$, but several candidates either introduced the incorrect sign from vector $\overrightarrow{FN}$ or more commonly omitted to include the actual lengths of AM, MB and FG within their calculations. Hence $AM = \frac{1}{3}$, $MB = \frac{2}{3}$ and $FN = \frac{1}{2}$ were usually seen instead of $AM = 1$, $MB = 2$ and $FN = 1$, respectively.

(b) The main errors here resulted from candidates using their incorrect vectors from **(a)** or having the parameter λ attached to the vector $\overrightarrow{OM}$ instead of the vector $\overrightarrow{MN}$. Candidates should be reminded to give their answer as a vector equation, as required by the question. Candidates were not awarded the accuracy mark unless the left side of the equation was the vector **r**, stating that this vector was the line $\overrightarrow{MN}$ was insufficient.

(c) Most candidates found this part to be particularly challenging, mainly since they did not identify that it was necessary to find the vector $\overrightarrow{DP}$, where point P is a general point on the line found in **(b)**. Most candidates only considered vector $\overrightarrow{OP}$ or vector $\overrightarrow{OD}$. Candidates who realised it was $\overrightarrow{DP}$ that was required often combined $\overrightarrow{OP}$ and $\overrightarrow{OD}$ incorrectly. Having obtained vector $\overrightarrow{DP}$ the two methods available were to either find the minimum distance of DP, which only very few candidates opted for, or the more common approach of taking the scalar product of vectors $\overrightarrow{DP}$ and $\overrightarrow{MN}$ and equating the result to zero. In many cases a variety of other vectors, such as vectors $\overrightarrow{OM}$ and $\overrightarrow{ON}$ were chosen instead of the vector $\overrightarrow{MN}$. Amongst those that did reach a correct parameter value, and obtained the correct point on the line, some took vector $\overrightarrow{DP}$ to be their final answer instead of vector $\overrightarrow{OP}$.

Question 9

(a) Most candidates achieved full marks for this question. Candidates who opted for the more algebraic approach of solving three equations in three unknowns, which is typically more challenging, almost always produced a correct solution.

(b) Most candidates scored at least three marks in this question, with the most common error usually occurring in the term $\frac{1}{2+x}$, where candidates obtained $\frac{2}{1+\frac{x}{2}}$ and $\frac{\frac{1}{2}}{1+x}$ instead of $\frac{\frac{1}{2}}{1+\frac{x}{2}}$.

Question 10

(a) Candidates found this question to be challenging with many confusing their ideas and approaches, for example, many candidates opted to substitute $x + \mathrm{i}y$ for both v and w. Many other candidates having solved the linear equations to reach an equation in v or in w, for example $-w + \mathrm{i}w = 5 + 7\mathrm{i}$, treated w as real and took real and imaginary parts, resulting in $w = -5$ or 7. Such solutions were produced by the majority of the candidates. The correct approach was to either solve to obtain $w = \frac{5+7i}{i-1}$, or the equivalent expression for v, then multiply the numerator and the denominator by $(\mathrm{i} + 1)$ or substitute $w = a + \mathrm{i}b$ and then take real and imaginary parts. An alternative approach was to substitute $v = x + \mathrm{i}y$ and $w = a + \mathrm{i}b$ into the original equations and to take real and imaginary parts prior to any elimination of variables, resulting in four equations in four unknowns. From here these equations were easy to solve. Some candidates were successful in following either of these approaches.

(b) (i) Many fully correct solutions were seen. However the centre of the circle was often at (–2, 3), (–2, –3) or (2, –3). As mentioned in Key Messages it is essential that the scales of an Argand diagram are equal on both axes. It was often unclear that the radius was 1, this needs to be clearly shown with scales on both the axes so that the points (2, 2), (2, 4), (1, 3) and (3, 3) can be clearly identified.

(ii) Candidates found this question to be challenging, with few candidates using the correct method. Most candidates obtained the arg of the centre of the circle but did not continue to answer the question fully. A common error noted was the confusion of where the right angle was in the triangle involving the tangent, the radius and the line from the origin to the centre of the circle saw tan of the angle at the origin in this triangle taken as $\frac{1}{\sqrt{13}}$ instead of this value being the sine of the angle.

MATHEMATICS

Paper 9709/42
Mechanics

Key messages

- Non-exact numerical answers are required correct to 3 significant figures as stated on the front of the question paper. Cases where this was not adhered to were seen in **Question 5(a)** and **Question 5(b)**. Candidates would be advised to carry out all working to at least 4 significant figures if a final answer is required to 3 significant figures.

- When answering questions involving any system of forces, a well annotated force diagram could help candidates to make sure that they include all relevant terms when forming an equilibrium situation, a Newton's Law equation or a work-energy equation. Such a diagram would be particularly useful in **Question 1(b)**, **Question 2(b)**, and **parts (a)** and **(b)** of **Question 6**.

- In questions such as **Question 7** in this paper, where displacement is given as a function of time implying that acceleration is not constant, it is important to realise that calculus must be used and that it is not possible to apply the equations of constant acceleration.

General comments

The paper was generally very well answered by many candidates although a wide range of marks were seen.

The presentation of the work was good in most cases; however candidates are reminded to write answers clearly using black or dark blue pen.

In **Question 2**, the angle α was given exactly as $\tan\alpha = \frac{3}{4}$. In cases like this there is no need to evaluate the angle and problems such as this can often lead to exact answers and so any approximation of the angle can lead to a loss of accuracy.

The examination allowed candidates at all levels to show their knowledge of the subject, whilst differentiating well between even the stronger candidates. **Questions 1(a)**, **2(a)** and **3(a)** were found to be the most accessible questions whilst **Questions 4**, **6** and **7** proved to be the most challenging.

One of the points on the front of the question paper tells candidates to take *g* = 10 and it has been noted that nearly all candidates are following this instruction. In some cases, it is impossible to achieve a correct given answer unless this value is used.

Comments on specific questions

Question 1

(a) This question was completed successfully by almost all candidates. The definition of Power as rate of doing work must to be used and in this case, it takes the form 'Power = $\frac{\text{Work Done}}{\text{Time}}$' to give the answer. In this problem, the power is expressed as $P = \frac{750\,000}{10}$. A common error that was seen was when candidates multiplied the work done by 10 rather than dividing. A few candidates gave their final answer using incorrect units.

(b) Most candidates made a good attempt at this question. The driving force is obtained using the relationship $\text{DF} = \frac{P}{v}$ where *P* is the power found in **Question 1(a)**. Newton's second law can then be used with the forces acting; the driving force, DF and the resistance force, 2400 N. The required acceleration, *a*, can then be found using the equation DF – 2400 = 16 000*a*. Some errors seen were to forget to include the resistance force or to have the incorrect sign on this resistance force.

Question 2

(a) Most candidates made a very good attempt at this question. Almost all used the equation $s = ut + \frac{1}{2}at^2$ with $u = 0$, $a = 2$ and $s = 1.44$ to find the required time. An alternative method used by some was to find the velocity, *v*, when the particle had moved through a displacement $s = 1.44$ using the equation $v^2 = u^2 + 2as$ which gives $v = 2.4$, from which the required time can be found. Very few candidates did not find this time correctly.

(b) The best approach to this question is to resolve forces vertically which enables the normal reaction, *R*, to be found and then to resolve forces in the direction of motion which gives the value of the friction force, *F*. Once these two values are found then the coefficient of friction, *μ*, is found using $\mu = \frac{F}{R}$. This was the approach taken by most candidates. However, a common error seen was to take the normal reaction as $R = 4g$, not taking into account the component of the 3 N force. Sign errors and trigonometric errors in the two equations also lead to some incorrect answers.

Question 3

(a) This question was well answered by most candidates. The initial kinetic energy must be evaluated and an expression formed for the gain in potential energy. As there is no resistance to motion the energy equation takes the form, loss in kinetic energy is equal to the gain in potential energy. This leads to the required result. However, some candidates incorrectly assumed that it was possible to use the equation $v^2 = u^2 + 2as$ to find the height. Since the motion is not in a straight line it is not possible to use constant acceleration equations. Whilst a few candidates chose this incorrect method, the majority of candidates found the required value of *h* correctly.

(b) This question proved to be challenging for a large number of candidates. Many issues occurred because candidates incorrectly assumed that the speed of the particle at *B* was 3 ms^{-1} as it was in **part (a)**. However, this value only applied in **part (a)**. As with all energy problems it must first be decided from which point to measure the zero level of potential energy. If this zero level of PE is taken as the level of point *C*, then the total initial energy is $\frac{1}{2} \times 0.2 \times 5^2 + 0.2 \times 10 \times 0.5$. If *v* is the speed of the particle at *C* then the work-energy equation states that the total initial energy must be equated to $\frac{1}{2} \times 0.2 \times v^2 + 3.1$. Some candidates incorrectly tried to use the given 3.1 to enable a friction force to be found. Others assumed that the particle reached *B* with the same speed as in **part (a)** and then used the 3.1 only on the section *BC*. Another error was to not include all of the relevant terms in the work-energy equation.

Question 4

(a) This question was challenging for many candidates. One method of approach is to find expressions for the distances *AB* and *BC* in terms of the required acceleration, *a*. Although it was straightforward to find the distance *BC* by using the equation $s = ut + \frac{1}{2}at^2$, it was clear that many candidates were unfamiliar with the equation $s = vt - \frac{1}{2}at^2$ which was the most straightforward method of finding the distance *AB*. Once the values of *AB* and *BC* were found, use of the given information that $AB = \frac{4}{5}BC$ enabled the required acceleration to be found. Many different approaches were seen. These often included use of the speed, *u*, at *A*, such that $AB = 2u + 2a$ and the equation $u = 4.5 - 2a$. This is a perfectly acceptable method and also leads to the correct value of *a*. However, a significant number of candidates incorrectly thought that $u = 0$.

(b) In this part of the question use of the acceleration found in **part (a)** enabled the required distance to be found. A number of candidates had already found the value of *AB* and/or *BC* in their calculations for **part (a)** and so merely had to combine these answers to produce their result in this part. Most candidates attempted to use their acceleration value found in **part (a)**.

Question 5

(a) This question states that the resultant of the four forces is in the direction of the 3 N force. This means that the resultant force acting perpendicular to the 3 N force is zero. By resolving forces in this direction and setting this to zero gives the required value of *F*. Most candidates solved the problem correctly. However, some candidates incorrectly resolved forces parallel to the 3 N force and set this to zero which gave an incorrect value for *F*. Another error seen was where candidates incorrectly resolved in both directions, found two values of *F* and combined them using Pythagoras. There were also some sign errors seen and a mix up of sine and cosine when resolving.

(b) The standard method of solution to this problem is to resolve forces vertically and horizontally; this method was used by the majority of candidates. These two equations can be rearranged to give expressions for $F \sin \alpha$ and $F \cos \alpha$. There are various methods for solving these equations for *F* and α. One method is to square and add the two expressions which leads to a value of *F* and then α is found by substitution. Alternatively, dividing the two expressions leads to a value of $\tan \alpha$, from which α can be found and substitution leads to the value of *F*. Most candidates made a good attempt at this question. One common error seen was to attempt to use Lami's theorem, but this only applies to a system of three forces. Some sign and trigonometric errors were seen in some of the calculations. Some candidates did not retain enough significant figures in their working and so lost accuracy in their final answers.

Question 6

(a) In order to determine the force in the tow-bar, the best approach is to consider the motion of the trailer. The only forces acting on the trailer are the force in the tow-bar and the given resistance of 200 N. By applying Newton's second law to the trailer and using the given acceleration of $-12\ \text{ms}^{-2}$ then the value of the force T N in the tow-bar can be found. Many candidates used this approach but made sign errors when finding the value of *T*. It is clear from the result that the force in the tow-bar is a thrust. The question asked for the magnitude of this force and several candidates gave their answer as a negative number. Another common error seen was to apply Newton's second law to the system of car and trailer but the force in the tow-bar does not appear in this equation. One more common error was to apply Newton's second law to the car, but there is an unknown braking force also acting on the car and so it is not possible to find *T* directly by this method. A longer method of approach is to use the equation for the system and also the equation for the car. Both of these equations involve the braking force which could be eliminated between these two equations giving a value for *T*.

(b) In this part of the question the braking force is required and there are two possible methods of approach. Newton's second law can be applied either to the system of car and trailer or it can be applied to the car. If it is applied to the car, then the value of *T* found in **part (a)** must be used. If it is applied to the system then the only forces acting are the given resistance of (600 + 200) N and the required braking force. There were several cases seen where terms were missing depending on which method was used and many candidates made errors in the sign of the forces. Candidates would be advised to draw a simple diagram showing all of the forces acting on whichever body is being considered.

(c) In this part of question the initial speed $u = 22$ is given, as well as the displacement $s = 17.5$ and the acceleration $a = -12$. By using the equation $v^2 = u^2 + 2as$, the speed, *v*, of the car as it hits the van can be found. Almost all candidates found the required speed correctly. It must be remembered that as this is a given answer, care must be taken to show all of the working in such a case.

(d) This question involves the use of the principle of conservation of momentum. Before the collision the total momentum of the system is $(1600 + 700) \times 8$ by using the result from **part (c)**. If M kg is the mass of the van then the total momentum after the collision is $(1600 + 700) \times 2 + 5M$. Equating momentum before the collision to momentum after the collision enables the value of M to be found. Some common errors seen were to only include the car and not the trailer in the calculations. Another common error was to only consider the momentum after the collision. Some candidates incorrectly used the initial speed as $u = 22$ rather than $u = 8$.

Question 7

(a) This part refers only to the time up to $t = 6$. This requires differentiation of the given expression for s in order to find the velocity during this time and then setting this velocity to zero to determine the time at which the particle is at instantaneous rest. Almost all candidates correctly found this time although some candidates found the times when $s = 0$, rather than when $v = 0$.

(b) In order to find the velocity as the particle arrives at the point P, the value of $t = 6$ must be used in the expression found for velocity in **part (a)**. In order to find the velocity with which the particle leaves P, the expression for s given for the period $t \geqslant 6$ must be differentiated and this expression must be evaluated at $t = 6$. Most candidates found the velocity with which the particle arrives at P correctly. Some errors were seen in the differentiation of s for $t \geqslant 6$ and since the velocity was required, the final answer must be given with a negative sign to indicate the direction of motion but some candidates gave the answer as positive.

(c) Candidates found this question to be quite challenging. A sketch of the displacement-time graph for the period $0 \leqslant t \leqslant 6$ shows that the particle returns to the origin O at $t = 1$, comes to instantaneous rest at $t = 1.5$ and is again at O at time $t = 2$. This means that in order to find the distance travelled the journey must be divided into three time periods, namely, $0 \leqslant t \leqslant 1.5$, $1.5 \leqslant t \leqslant 6$ and $t \geqslant 6$, otherwise there will be a confusion between distance travelled and displacement. Using the given expression for s in the period $0 \leqslant t \leqslant 6$ it can be seen that at $t = 0$, $s = 2$ and that at $t = 1.5$, $s = -0.25$. Hence the distance travelled by the particle between the times $t = 0$ and $t = 1.5$ is 2.25 m. Since $s = 20$ at $t = 6$, the distance travelled between $t = 1.5$ and $t = 6$ is 20.25 m. Finally using the expression for s in the period $t \geqslant 6$ it can be seen that at $t = 10$, $s = 2.4$ and so the distance travelled between $t = 6$ and $t = 10$ is $20 - 2.4 = 17.6$ m. Adding these three distances together gives the total distance travelled. Many candidates did not consider the fact that the particle came to rest at a negative value of s and considered the region from $t = 0$ to $t = 6$ without reference to the time $t = 1.5$. This meant that they were not correctly finding the distance travelled during this time. Some candidates incorrectly integrated the expression for s in their attempt to find the distance.

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Probability and Statistics

Key messages

Candidates should be aware of the need to communicate their method clearly. Simply stating values often does not provide sufficient evidence of the calculation undertaken, especially when there are errors earlier in the solution.

Candidates should state only non-exact answers to 3 significant figures and exact answers should be stated in their exact form. It is important that candidates work to at least 4 significant figures throughout to justify a 3 significant figure value. The only exception is if a value is stated within the question. It is an inefficient use of time to convert an exact fractional value to an inexact decimal equivalent. There is no requirement for probabilities to be stated as a decimal.

General comments

Although many well-structured responses were seen, some candidates did not use the response space in a clear manner which made it difficult to follow their thinking within their solution. Where a solution is deleted, the use of the **Additional Page** was the most effective process to provide the replacement solution if there is insufficient space remaining, however a significant number of candidates used the space either side of their original attempt resulting in poor clarity.

The use of simple sketches and diagrams can help clarify both context and conditions. These were frequently present in good solutions.

Many good solutions were seen for **Questions 1**, **5** and **6**. The context in **Questions 4** and **7** was found challenging by many. Sufficient time seems to have been available for candidates to complete all the work they were able to, although a few candidates did not appear to have prepared well for all topic areas of the syllabus.

Comments on specific questions

Question 1

Almost all candidates recognised this as a selection question and used combinations within their solutions. The best solutions had a simple narrative to clarify the context and identify how many members needed to be considered for travelling in the coach. The most common errors were using either 40 or 39 as the number of members who were available to travel on the coach which ignored the travel requirements for Ranuf and Saed. A significant proportion of solutions combined their value for coach travel selections with a value for travelling in the car, some correctly identified that there was only one possible selection once the coach had been filled whereas many calculated an alternative value using a further combination approach. Although the majority then multiplied their values as anticipated, a few summed their values. It was noted that almost all candidates did not round their exact value answer in accordance with the general paper instructions.

Question 2

Almost all solutions used methods appropriate for Discrete Random Variables in this question. A small number of solutions worked with decimals, which is less efficient and less accurate than using the fractional approach.

(a) The majority of candidates recognised that a geometric distribution was appropriate for this context. The most successful solutions simply calculated the probability for the required outcomes and summed the results. A few candidates used $\frac{1}{6}$ as the success probability, which does not allow that either 1 or 6 would finish the activity. Some candidates used the alternative approach of finding the difference between needing at least 3 and at least 6 throws, however many of these candidates confused the power value that was required within their formula. Weaker responses assumed that this was a binomial distribution and gained no credit.

(b) Almost all candidates who attempted this question presented their solution as a probability distribution table. Weaker responses were not always successful in identifying the correct outcomes, often omitting 0 and sometimes including 4 etc in their table. Better solutions included separate workings to justify the table values, clearly indicating the binomial nature of the distribution. Weaker solutions did not apply fully the binomial distribution for outcomes 1 and 2, leading to a probability distribution table which did not total to 1. Some candidates used decimal equivalents within the table, where rounding to 3 significant figures resulted in a probability of less than 1. It is good practice to use exact values whenever possible.

(c) Almost all solutions used the appropriate approach with the values in the probability distribution table in **part (b)**. Better responses clearly stated the calculations that were necessary and then efficiently evaluated with no further working. Some weaker responses showed attempted use of $E(x) = np$.

Question 3

All but the weakest solutions used the normal standardisation formula appropriately, although there were inaccuracies noted in finding the *z*-value associated with the probability within the context. The majority of candidates correctly identified that as weight is a continuous variable, no continuity correct was required.

(a) Better solutions often included a simple diagram to clarify the context. There was usually a clear statement of an equation involving the normal standardisation formula and a *z*-value, although different probabilities were noted and the *z*-value was not always accurate. Candidates should be aware that stating a final answer as 6.5 is not acceptable, since non-exact answers must be stated to 3 significant figures.

(b) The most successful solutions often included a simple diagram to clarify the context. It was anticipated that candidates would realise that the information provided in the question was the value required for the numerator in the normal standardisation formula. However, the majority of candidates calculated appropriate values of 86 grams and/or 78 grams and then substituted into the formula. A number of arithmetical errors were noted at this stage. A small number of candidates did not use the mean stated in the question or used a numerator of 8 in their standardisation formula.

Better solutions recognised the symmetrical nature of the context and having worked out one probability use this symmetry to state the other required probability. Weaker solutions simply worked out the probability that the apple weighed more than 78 grams.

Question 4

The majority of candidates recognised that as the candles were identical, repeats needed to be removed and there was some process to undertake this in most solutions.

(a) Although this was a simple context, many candidates misinterpreted the information and assumed that the red candles were not identical. Better solutions included a simple diagrammatic representation of the context for clarity and included simple explanations. Weaker responses often only removed the repeats for one colour candle or calculated the number of arrangements if all candles were different.

(b) Although many candidates appeared to find the context of this question challenging, a large number of good solutions were seen and a number of different approaches were taken. Many solutions included a simple diagram to represent the blue and green candles to identify where the red candles could be placed. The most efficient solution was to consider that there were seven items which were not red candles and then determining that there were ${}^8C_2$ ways of placing the two red candles, although ${}^8P_2$ was seen frequently. A significant number of candidates did not apply the fact that the green candles were identical and so the repeated arrangements needed to be eliminated.

A large number of candidates attempted to calculate from the total number of arrangements with just the blue candles together and subtract the number of arrangements of blue candles together and red candles together. This approach tended to be less successful largely due to the blue candles not being kept as a block in calculations. In this approach, almost all candidates did eliminate the repeated arrangements for the green candles in every term.

Question 5

Almost all candidates identified that the question related to probability using the binomial distribution. Candidates were often unsure in their interpretation of the condition requirements within the contexts and the appropriate boundaries were not often used.

(a) The most common, and efficient, approach was to subtract the probabilities of the outcomes not required from 1. Better solutions clearly stated an unsimplified expression using binomials terms and appropriate use of brackets or operations before using the calculator efficiently to state the answer. Many candidates evaluated individual terms initially, which is not expected at this level. This also resulted in some loss of accuracy with premature approximation or inaccurate rounding being noted. A significant number of solutions included 6 as a required outcome, which is excluded by the requirement of 'less than 6'.

A few candidates used the less efficient approach of adding the 6 outcomes that fulfilled the requirement.

(b) Nearly all candidates identified that the normal distribution was an appropriate approximation for the context. Better solutions included a formal check initially to confirm before proceeding with the question. Most candidates recognised that a continuity correction was required in the standardisation formula because the variable was discrete. Good solutions stated unsimplified calculations for the mean and variance before substituting into the standardisation formula and evaluating the *z*-value. The inclusion of a simple sketch helped to clarify the required probability area. There was some evidence of premature approximation of the z-value and candidates should be reminded that working to at least 4 significant figures is the expectation until the final answer, if not exact.

Question 6

Many candidates found this probability question accessible and provided good evidence of their calculations throughout.

(a) Although many full correct tree diagrams were seen, a significant proportion did not include placing the ball that was picked from Box *A* into Box *B* before picking the ball from that box. Almost all solutions used fractions for probabilities, those that converted to decimals usually stated to 3 significant figures which is acceptable here but led to premature approximation errors later in the question. A number of solutions had branch pairs which did not total to 1.

(b) The best solutions stated the colour combinations that were required and then stated an unsimplified expression using the values from their tree diagram, which was evaluated efficiently. Weaker solutions simply stated the two outcome probabilities which did not provide sufficient evidence of process if the tree diagram was inaccurate. Candidates should be reminded that their workings need to communicate clearly their planned process even if their final answer is correct.

(c) Most solutions clearly communicated the use of the standard conditional probability formula. Better solutions also included a general statement of the formula before values were substituted. Weaker solutions only considered one branch using $\frac{1}{8}$ in the numerator.

Question 7

Many candidates found this question on representation of data challenging. The question was in a real-life context, so some standard processes needed to be adapted to ensure that fish did not have a negative length, which is impossible.

(a) Almost all candidates attempted to draw a cumulative frequency graph, although histograms and frequency polygons were also noted. Good solutions stated the cumulative frequency before drawing the graph. Many candidates efficiently used the data table, with the best extending the table with an appropriate narrative. The best solutions used a vertical scale that allowed the values to be plotted accurately, identified the correct boundaries of the classes for plotting the cumulative frequencies, fully labelled both axes and drew a smooth curve. Candidates should be reminded that axes involving variables should include units when labelled. Although only a small number of candidates plotted at mid-values, there was inconsistency in interpreting the upper boundary of the classes, with many not allowing for the discrete nature of the data and plotting at the stated values. This was particularly noticeable with the final class which was often plotted incorrectly even after previous correct plots at 29.5 or 30. As real-life data was used, it was anticipated that the curve would start at (0,0) rather than (–0.5, 0) which is an invalid length for a fish.

(b) Many candidates misinterpreted this question and used their graph to estimate the length which at least 40 per cent of the fish have. Good solutions identified that 60 per cent of the fish would have a length of less than *d* cm, calculated the value and drew lines on their graph to show how the estimate was achieved. Weaker solutions simply provided a value with little evidence that it had been read off the graph. Where a question specifies a technique must be used, candidates would be well advised to ensure that this is communicated clearly within their work. A number of solutions were seen which used a sample size of 160 in this question. It was noted that candidates that had used a more complex scale in **part (a)** were less successful in stating their value accurately.

(c) While the majority of attempted solutions used the correct variance formula, many did not apply the real-life context appropriately and found the theoretical mid-value of the first class rather than realising that fish could not have a negative length. The best solution provided a variance formula with all the values substituted which was then evaluated with no further working shown. Many good candidates included an expanded data table and included all the necessary values within this structure efficiently. The most common error was using 4.5 as the mid-value for the first class. A number of candidates recalculated the mean length of the 150 fish, not always achieving the value given.

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Probability and Statistics

Key messages

- If candidates use the Additional Page for working, it is important that the question number is clearly indicated.
- It is important that candidates recognise the need to round answers to the required number of significant figures, this was particularly apparent in **Question 1**.
- Candidates are always reminded to provide full and clear method to support their answers. Instances where candidates were not doing this was in **Questions 1** and **7**.
- When carrying out a significance test the conclusion must be in context and not definite, this was particularly important for **Question 3(b).**

General comments

Candidates appeared to find this paper reasonably accessible, with some responses being to a very high standard.
Questions 3(a) and **4(a)** were particularly well attempted, as was **Question 6**. Questions that proved to be more challenging were **Questions 5** and **7**.
It is important that candidates can round correctly to three significant figures. There were instances in candidate responses where only two, or even one significant figures were given with no indication of more accurate figures; this would result in a loss of accuracy.
There did not appear to be any time issues for candidates on this paper, and presentation was generally good.
The comments below indicate common errors and misconceptions, however, there were also many full and correct solutions presented too.

Comments on specific questions

Question 1

A large number of candidates used the correct method to find the required probability, though rounding and premature approximation errors were common. The value for λ was 5/12; many candidates used 0.417 or 0.4166 (both acceptable) but a value of 0.42 or 0.416 was not. The final answer of 0.0661 or 0.0662 was often given to only 2sf (0.066), this was not acceptable unless more accurate figures were seen before rounding. This error in rounding is probably due to a confusion between significant figures and decimal places; it is important that candidates distinguish between these two ways of rounding. The accuracy required on this paper is to at least 3sf, so an answer here of 0.066, alone, is insufficient.
A few candidates attempted to use a Binomial distribution, rather than a Poisson, which was often unsuccessful and was not as required by the question. It was important that candidates gave full method to support their final answer of 0.0661 by using a Poisson distribution which was the 'suitable distribution' as required by the question.

Question 2

Many candidates were able to find the correct equation connecting the width of the confidence interval to the z value, although there were occasional missing factors of 2. Most candidates then went on to find the correct z value of 1.953 and then successfully looked this up in the tables to find 0.9746. However, at this point many candidates were unable to then find the correct value for α and either thought it was 97.46%, or attempted an incorrect method to find α. It was again important that clear working was shown from finding the z value of 1.953 to reaching the final value for α.

Question 3

Part (a) was particularly well attempted. Most candidates correctly found estimates for the population mean and variance with very few candidates calculating the biased variance rather than the unbiased. There were only a few cases seen where candidates confused the two formulae for the unbiased variance.
Part (b) was also well attempted; though some candidates omitted to give hypotheses or did not give them precisely enough. The comparison of *z* values, or area, needed to be fully shown, and the final conclusion needed to be in context, and not definite. Many errors were made when attempting the comparison and in some cases it was omitted completely. Concluding statements such as ‘the mean time has decreased’ or ‘there is evidence to reject Ho’ would not be acceptable; the first statement being definite as it is saying that we know for certain that the mean time has decreased and the second is without context.

Question 4

Part (a) was very well attempted. Many candidates found a correct value for λ, and used it to find the required probability. Errors included calculation of P(0, 1, 2, 3) rather than P(0, 1, 2) and use of an incorrect λ.
In **part (b)**, a good attempt was made by some candidates to find *n*. Many realised that the inequality they found could be solved by taking log (base e). Reaching 6.411 was often successfully done, but many candidates then left this as their final answer, or incorrectly concluded that *n* was 7. Other errors included premature approximations, using –0.05 instead of –0.05129 for ln 0.95 as well as unsuccessful attempts at a trial and improvement method seen.

Question 5

Candidates generally found this to be a challenging question; **part (c)** indicated lack of understanding and interpretation of the context given in the question in some cases.
In **part (a)**, many candidates successfully integrated f(*x*), but many did not use correct limits. Some candidates tried to find E(*X*) and Var(*X*) and some went on to use an invalid normal distribution.
In **part (b)**, many candidates attempted to set up a correct equation involving 0.25 (or 0.75) but the limits used were not always correct, and some candidates did not declare any limits at all. Those who used a correct equation and correct limits were, on the whole, able to use correct algebra to reach the answer given.
Part (c) was poorly attempted with some candidates omitting this part completely. Many candidates incorrectly thought that *q* would be twice the value of *p*, and some attempted to integrate f(*x*) between –*q* and +*q* and equate to 0.5, although few candidates were successful in doing so. This led to the equation found in **(b)** but in *q* rather than *p* thus a clear deduction that *q*=*p* could be made. A quicker and more straightforward method was to use the symmetry of the curve to deduce that *q*=*p*; use of a diagram could have helped candidates here.

Question 6

This question was well attempted with many candidates using the correct method to find the probabilities required. However, not all were successful in finding the correct values for the mean or, more often, for the variance. Standardising and use of tables was generally done well in both parts. Candidates should however be reminded that working should be fully shown. Common errors included the inclusion of an unrequired continuity correction, and use of the wrong area when finding the required probability; candidates would be advised to sketch a diagram.

Question 7

This proved to be a demanding question for a large number of candidates. In **part (a)** some candidates found the rejection region from invalid methods (for example, finding individual probabilities and not a tail probability), and some gave unsupported answers for the probabilities needed to justify the rejection region. Working out must be shown so that it is clear that B(20, 0.95) was used to find $P(X \leqslant 17)$ and $P(X \leqslant 16)$; merely quoting B(20, 0.95) was not sufficient. These probabilities should have been worked out to three significant figure accuracy but often were only given to two significant figures. Some candidates only found $P(X \leqslant 16)$, which was not sufficient to fully justify the rejection region. Weaker responses attempted incorrect solutions using a Normal distribution.
In **part (b)** many candidates incorrectly thought that the probability was 0.02 (from the level of significance).

Some candidates correctly used B(20.0.7) in **part (c)** to find the probability of a type II error, but there was often confusion over which region related to the type II error. As in **part (a)** incorrect use of a Normal Distribution was often seen.